Comment on "Hypersonic Wakes and Trails"

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LEES concludes "that at hypersonic speeds, transition in the wake of a blunt body cannot occur aft of $(x/d) \cong 40$."[1] To arrive at this conclusion, Lees combines an $(Re_{f,\,d})_{\min,\,cr}$ from cylinder experiments at Jet Propulsion Laboratory (JPL) (Ref. 35)† with an $(Re_{x,\,f})_{TR} = 56{,}000$ derived from both cylinder experiments at Graduate Aeronautical Laboratory, California Institute of Technology (GALCIT) (Refs. 1 and 8) and from sphere experiments at Lincoln Laboratory (Refs. 7 and 9): $(x/d)_{\min,\,cr} = (Re_{x,\,f})_{TR}/(Re_{f,\,d})_{\min,\,cr} \cong 40$.

The farthest point downstream at which wake transition can occur is taken to be $(x/d)_{\min,\,cr}$. Beyond this point, according to Lees' reasoning,[1] wake turbulence cannot maintain itself against the action of viscous dissipation; roughly speaking, at this point, $\bar{\epsilon}_T \approx \nu_f$. This reasoning seems superficially appealing; but further thought reveals that the criterion is conceptionally more applicable to the location of the *quenching* of the turbulent motion and not to the location of inception. Inception involves linear and then nonlinear amplifications of small disturbances and does not involve $\bar{\epsilon}_T$.

Figure 11a of the original paper is reproduced here as Fig. 1. This shows the data points and the correlation $(Re_{x,\,f})_{TR} = 56{,}000$. Added to the original figure are data points from Ref. 7 for $(x/d) \geq 35$. The numerical values for each of these new data points is taken from Demetriades and Gold,[3] as is the case for all of the original data points in Fig. 1. Two things are to be noted. First, $(x/d)_{TR}$ can be >40 as was presented in Ref. 7, Fig. 3. Furthermore, in their paper in this issue of the AIAA Journal, Clay, Labitt, and Slattery[2] show that $(x/d)_{TR}$ can be $\gg 40$ (see Fig. 6). Second, the correlation $(Re_{x,\,f})_{TR} = 56{,}000$ is for data points $(x/d)_{TR} \leq 35$, as noted in the original legend of Fig. 1. It is obvious that the $(Re_{x,\,f})_{TR}$ correlation is not valid for the transition point moving off to distances far downstream. Hence, it is inappropriate to use $(Re_{x,\,f})_{TR}$ to obtain an $(x/d)_{\min,\,cr}$, since this correlation is invalid in the regions of $(x/d)_{\min,\,cr}$. It is likely that the $(Re_{x,\,f})_{TR}$ correlation is valid only in the region of strong favorable pressure gradients and compressibility effects which, for blunt bodies and $M_\infty \lesssim 10$, is in the region $(x/d) \leq 35$. Goldburg[4] has suggested that a second correlation must be obtained in the downstream region where pressure decay and compressibility effects are small:

$$(Re_{f,\,d})_{TR}\cdot(d/x)_{TR} = [(d/x)_{TR}]_{\substack{\text{upstream}\\ \text{limit of second}\\ \text{correlation}}} \times \{(Re_{f,\,d})_{TR} - (R_{\,f,\,d})_{\min,\,cr}\}$$

From the sphere data of Fig. 1 (Refs. 7 and 9), the second correlation leads, in the asymptotic limit of $(x/d)_{TR} \to \infty$, to an $(Re_{f,\,d})_{\min,\,cr} = 6000$, $M_\infty = 3$ to 10, based on average local properties along the wake front in the downstream regions. This value agrees with the $(Re_{f,\,d})_{TR}$ of the Lincoln Laboratory data at $(x/d)_{TR} \approx 100$, and with the $(Re_{f,\,d})_{TR}$ derived from the Avco-Everett Research Laboratory experiments.[5]

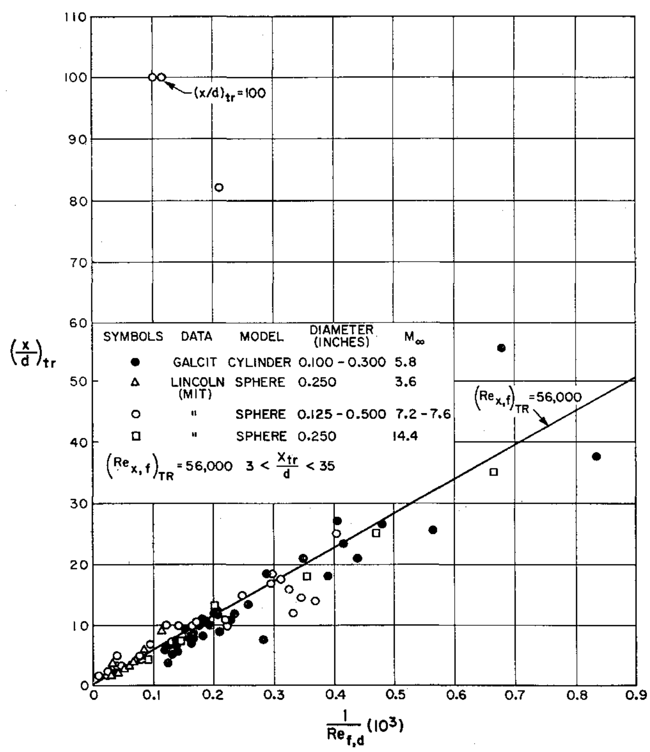

Fig. 1 Correlation of transition in hypersonic wake behind blunt bodies; Fig. 11a of the original paper[1] with additional data points for $(x/d)_{TR} \geq 35$.

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† The notation and the references are the same as in the original paper.

References

[1] Lees, L., "Hypersonic wakes and trails," AIAA J. **2**, 417–428 (1964); see Sec. 3.

[2] Clay, W. G., Labitt, M., and Slattery, R. E., "Measured transition from laminar to turbulent flow and subsequent growth of turbulent wakes," AIAA J. **3**, 837–841 (1965).

[3] Demetriades, A. and Gold, H., "Correlation of blunt-bluff body wake transition data," Graduate Aeronautical Laboratory, California Institute of Technology, Hypersonic Research Project, Internal Memo 12 (September 20, 1962).

[4] Goldburg, A., "Analysis of hypersonic wake transition experiments," Avco-Everett Research Laboratory Research Note 391 (to be published).

[5] Fay, J. A. and Goldburg, A., "Unsteady hypersonic wake behind blunt bodies," AIAA J. **1**, 2264–2272 (1963).

Reply by Authors to A. Goldburg

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GOLDBURG has apparently misunderstood the significance of the local minimum critical Reynolds number $(Re_{f,\,d})_{\min,\,cr}$. When the effective turbulent diffusivity $\bar{\epsilon}_T$ is smaller than the kinematic viscosity ν_f, the work done by the Reynolds stresses is more than counterbalanced by the rate of viscous dissipation. Thus, turbulence in the wake cannot exist when $(\bar{\epsilon}_T/u_f d) < (\nu_f/u_f d)$, or when $Re_{f,\,d} < (Re_{f,\,d})_{\min,\,cr}$, any more than turbulent flow in a pipe can exist below a critical Reynolds number of 2000. If one chooses to call this "quenching," that is a problem in semantics and not in fluid mechanics. In Ref. 1, the value of $(Re_{f,\,d})_{\min,\,cr}$ was estimated roughly at about 500 for blunt bodies. Not unexpectedly, experiments on cylinder wakes by Kendall[2] and by one of us (Behrens, unpublished) showed that the actual value lies in the range 1200–1500, at least for two-dimensional wakes. None of the data on wake transition, whether new or old, shows any evidence of turbu-

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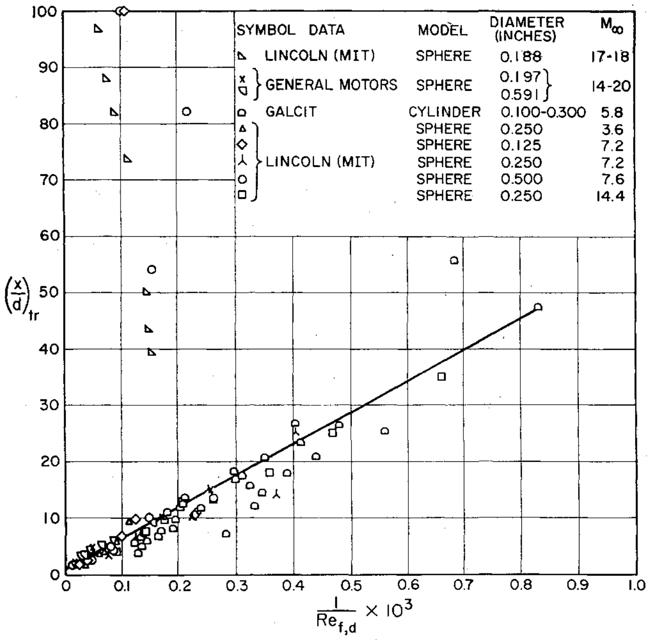

Fig. 1 Correlation of transition to turbulent flow in hypersonic wakes behind blunt bodies.

lence in the inner viscous wake or "core" at values of $Re_{f,\,d}$ below 1200 (Fig. 1).

The most rearward possible location of transition $(x/d)_{TR}$ in the "core" region depends not only on the value of $(Re_{f,\,d})_{\min,\,cr}$, but also on the value of $(Re_{x,\,f})_{TR}$, provided that such a unique transition Reynolds number exists. In Fig. 1, we show representative data on transition behind spheres and cylinders at hypersonic speeds from all available sources presently known to us. This data includes all the "old" data shown in Fig. 11a of Ref. 1; four "old" data points of Slattery and Clay[3, 4] at $M_\infty = 7.2$ and 7.6 not given in Ref. 1, but originally plotted by Demetriades and Gold,[5] and shown also in the preceding Comment; some of the new sphere data obtained by Clay, Labitt, and Slattery[6] at $M_\infty = 17$–18; new sphere data obtained by Wilson[7] in the Mach number range 14–20. Wilson's data fall along the same line $(Re_{x,\,f})_{TR} \cong 5.6 \times 10^4$ as most of the other results plotted in Fig. 11a of Ref. 1. Since his experiments were carried out with spheres 5 mm and 15 mm in diameter, the insensitivity of the value of $(Re_{x,\,f})_{TR}$ to body diameter is confirmed, at least to a distance of about 15 diameters behind the body.

On the other hand, the data of Clay, Labitt, and Slattery[6] at $M_\infty = 17$–18 lie well above the line $(Re_{x,\,f})_{TR} = 5.6 \times 10^4$, and are not in agreement with their data at $M_\infty = 14.4$ previously reported[3, 4] (Fig. 1). Their argument that these new results represent a "velocity effect" is weakened not only by Wilson's data, but also by their four points at $M = 7.2$ and $M = 7.6$ shown in Fig. 1, which also lie well above the "main line" of the data in the Mach number range 3.6–14.4. Thus we are confronted with an experimental contradiction, and it seems pointless to propose new "correlations" until this contradiction is resolved.

A possible explanation of these puzzling results is suggested by Clay, Labitt, and Slattery[6] and made more specific by Wilson.[7] At hypersonic speeds, the density field behind a blunt body is dominated by the "inviscid" wake generated by the bow shock. The radial boundaries of the steepest density gradients in the "inviscid" wake are clearly visible in schlieren photographs, especially for small bodies (e.g., Fig. 7 of Ref. 7), and these boundaries grow outward very slowly with axial distance. The laminar viscous wake near the axis also grows very slowly, and the density along the outer "edge" of this viscous core is 5–15 times lower than ambient density. At low pressures, transition may occur within this low-density region and not be visible. One may observe, instead, the emergence of the turbulent core into the higher-density zone much further downstream. Similar remarks apply to the growth of the wake width as measured by the UHF microwave cavity.[6]

Clay, Labitt, and Slattery[6] also report observations of transition to turbulent flow occurring 2000–6000 diameters downstream at $M_\infty = 17$–18 at low pressures. Recent measurements by one of us (Behrens) in the wake behind a circular cylinder in the Galcit hypersonic wind tunnel at $M = 6$ at $Re_{\infty,\,d} = 960$ uncovered evidence of a rapid increase in the rate of growth of the wake at a distance of 1600 diameters downstream of the body. This phenomenon may also represent transition to turbulence.‡ At such extreme distances behind the body and at these Reynolds numbers, the distinction between "inner" viscous wake and "outer" inviscid flow is no longer tenable. When the "core" goes turbulent near the body, this turbulent inner wake swallows the outer flow and a large-scale instability does not have time to develop. However, if $(Re_{f,\,d}) < (Re_{f,\,d})_{\min,\,cr}$, the entire flow between the wake axis and the bow wave is laminar for some distance behind the body, and this flow, too, is dynamically unstable.§ Since the velocity difference across this flow is quite small and the characteristic wavelength is large, one expects the amplification rate to be low. Thus, transition in this flow may occur far behind the body. Since the turbulent diffusivity in this region is based on the *total* drag of the body, the value of $(Re_{f,\,d})_{\min,\,cr}$ for this flow is correspondingly lower than the value of 1500 for the inner viscous wake near the body. Laminar stability theory[8, 9] may serve as a useful guide to an understanding of this interesting phenomenon.

References

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[9] Lees, L. and Gold, H. "Stability of laminar wakes," International Symposium on Fundamental Phenomena in Hypersonic Flow (June 1964).

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§ The sharp "break" in the curve of $X_{TR}/(C_DA)^{1/2}$ vs $P(C_DA)^{1/2}$ reported in Ref. 6 may represent the rapid downstream movement of transition in a narrow pressure range as $Re_{f,\,d} \rightarrow (Re_{f,\,d})_{\min,\,cr}$.